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LONGITUDINAL DISPERSION IN UNIFORM POROUS MEDIA

D. R. F. Harleman, et al

Massachusetts Institute of Technology

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

HYDRODYNAMICS LABORATORY

DEPARTMENT OF CIVIL ENGINEERING

REPORT NO. 57

LONGITUDINAL DISPERSION IN UNIFORM POROUS MEDIA

BY

D. R. F. HARLEMAN, P. F. MEHLHORN, and R. R. RUMER, JR.

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PREPARED UNDER
RESEARCH GRANT NO. WP-220(C1)
DIVISION OF WATER SUPPLY AND POLLUTION CONTROL
PUBLIC HEALTH SERVICE
DEPARTMENT OF HEALTH, EDUCATION AND WELFARE

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Department of Civil Engineering
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Cambridge 39, Massachusetts

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ACKNOWLEDGEMENT

This investigation was conducted at the Hydrodynamics Laboratory of the Department of Civil Engineering at the Massachusetts Institute of Technology under the sponsorship of the Division of Water Supply and Pollution Control, Public Health Service, Department of Health, Education and Welfare, Grant No. WP-220(C1). The study reported here was a part of an investigation of the dynamics and dispersion of salt water in porous media which was reported in Report No. 55.

This program was under the technical direction of Dr. Donald R. F. Harleman, Associate Professor of Hydraulics and Dr. Arthur T. Ippen, Professor of Hydraulics, who also acted as administrative supervisor for the Division of Sponsored Research of the Massachusetts Institute of Technology.

ABSTRACT

This report represents the results of an experimental investigation into the effects of the characteristics of a porous medium on the longitudinal dispersion of two miscible fluids flowing through it. The investigation was primarily concerned with the effects of grain size and shape, and the uniformity of the grain size distribution. Good correlation was achieved between the grain size and the longitudinal constant of dispersion, α_l, for uniform media consisting either of spherical or angular particles. It is shown that the longitudinal constant of dispersion, α_l, and the intrinsic permeability, k_p, which are functions only of the medium, are related by the expression $\alpha_l = (\text{const.})\sqrt{k_p}$, where the constant is a function only of the particle shape for uniform media. For angular particles the constant is 60 and for spherical particles the constant is 42, when k_p is in cm^2.

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LIST OF SYMBOLS

C = coefficient dependent upon pore geometry

d = 50 % grain size of medium [L]

D_1 = coefficient of longitudinal dispersion [L^2/T]

D_2 = coefficient of lateral dispersion [L^2/T]

D_M = effective molecular diffusivity in porous media [L^2/T]

g = acceleration of gravity [L/T^2]

h = piezometric head [L]

k_p = intrinsic permeability [L^2]

K = Darcy permeability [L/T]

n_1 = exponent which is dependent upon pore geometry

n_2 = exponent which is dependent upon pore geometry

R = Reynolds number

s = average concentration in a small but finite volume (treated as a point concentration)

t = time [T]

U = superficial velocity [L/T]

u,v,w = seepage velocity components in three coordinate directions [L/T]

x,y,z = three coordinate directions [L]

α_1 = coefficient dependent upon pore geometry (sometimes called dispersion constant or intrinsic dispersivity)

α_2 = coefficient dependent upon pore geometry

β = constant related to pore geometry

γ = specific weight of liquid [F/L^3]

η = D_1/ux

θ = porosity

μ = dynamic viscosity of liquid [FT/L^2]

ν = kinematic viscosity of liquid [L^2/T]

ξ = ut/x

ρ = density of liquid [FT^2/L]

Φ = $(1-\xi)/2\sqrt{\xi\eta}$

U.C. = uniformity coefficient (60 % grain size divided by 10 % grain size)

I. INTRODUCTION

The phenomenon of mass transport in liquid flow through a porous medium has been studied by investigators in many fields. Civil and sanitary engineers encounter many instances where this phenomenon plays an important role. The problem of sea water intrusion into coastal aquifers is becoming more and more acute as large coastal cities are beginning to experience the effect of increased ground water usage. Excessive pumping, perhaps many years ago, has paved the way for the intrusion of sea water inland to the point where fresh water supplies are now threatened.

As the sea water moves inland it disperses in a manner such that the saline water is carried ahead of the actual bulk flow. Sodium chloride in water at a concentration of only a few hundred parts per million is sufficient to render the water unfit for consumption. Oscillation of the inland flow of salt water by tides or periodic pumping only serves to increase the dispersion until the polluted water may fill a volume substantially greater than that of the actual volume of salt water that had moved inland through the aquifer. At present much attention is also being given to the problem of underground disposal of radioactive and sewage wastes.

The petroleum industry has done extensive investigating of the mass transport phenomenon in porous media for multiple phase flow, and is primarily responsible for what information is available on the dispersion of flow when fluids of different densities and viscosities are present.

It has been demonstrated mathematically and experimentally that the size and shape of the pores are the governing aspects in the phenomena of dispersion. The problem, however, is a difficult one to treat mathematically because of the complicated and varying pore geometries.

The two approaches that have met with the most success are -- statistical, based on a fundamental theorem in the theory of statistics which states that a great number of independent repititions will produce a probability distribution, and one involving the gross behavior of the fluid flow incorporating additional mass flux terms to account for the process of dispersion [8][5]. Both approaches lead to a convective-dispersion equation and to a coefficient of dispersion which relates the mass flux due to dispersion to the local average concentration gradient, seepage velocity, and pore geometry.

A few attempts have been made to correlate the dispersion coefficient with media characteristics [7][3][11], one of which is the present investigation being reported. Only moderate success has been achieved, possibly because of the inherent difficulties in isolating the effects of the various characteristics.

It has been well-established that the phenomenon of dispersion is a result of the tortuosity and intermingling of flow paths. It is known that the tracer concentration becomes normally distributed about a point which moves with the average seepage velocity in the absence of adsorption effects. As this front moves through the media the tracer is spread out further and further from this point of mean concentration. The extreme penetration of the traces is seen to move with a velocity greater than that of the mean seepage velocity. As a result, the amount of mixing observed is many times greater than can be accounted for by molecular diffusion.

Since a complete mathematical solution of the problem of dispersion has not been found, more and more attention is being given to the experimental phase of the problem. At present only qualitative predictions in regard to the value of the dispersion coefficient are available. The ability to predict the coefficients of dispersion for field conditions would be a significant achievement in the study of dispersion of contaminants introduced into the ground water. At present, field tests have been devised by which the permeability of underground strata can be measured. If the coefficients of dispersion could be reliably correlated with the coefficient of permeability, additional knowledge could be obtained from these field tests.

The purpose of the present report is to summarize the results of an experimental investigation into the effect of the characteristics of the medium on the coefficient of longitudinal dispersion and the relation between the coefficient of longitudinal dispersion and the coefficient of permeability. The investigation was performed in an attempt to supplement existing information and to indicate a path for future investigations.

II. THEORETICAL CONSIDERATIONS

The analytical development to follow assumes the flow is laminar, the fluid is incompressible, and the medium is saturated. In the potential flow treatment of ground water flows, the Darcy or superficial velocity is most commonly used. This superficial velocity is defined by the quotient of the actual discharge divided by the cross-sectional area of the porous media through which it is flowing. The actual velocity varies from point to point throughout the pore system of the media. In the flow net or potential flow analysis, the superficial velocity would be the quotient of a stream tube discharge divided by the cross-sectional area of the stream tube. For flow in porous media where the mixing of two fluids (dispersion) is important, the actual fluid velocities are the important velocities in the mixing process. Because of the complex boundaries, the actual velocities of the flow in porous media cannot be defined. The average of the actual velocities can be defined. It is the quotient of the actual discharge divided by the actual area through which the flow takes place. This average velocity is called the seepage velocity. It is equal to the superficial velocity divided by the porosity of the medium.

2.1 Darcy Equation

The Navier-Stokes equations apply to laminar flow in porous media. The difficulty is to define the geometry of the flow boundaries. It has been demonstrated [6] based on energy considerations that the superficial velocity, U, in flow through porous media is given by:

$$U = Cd^2 \frac{\gamma}{\mu} \frac{dh}{ds} \qquad (1)$$

where

C = a dimensionless constant for a given medium in place, and is a function only of the pore geometry.

d = representative grain diameter for the medium, generally regarded as the 50 % size taken from a standard sieve analysis, [L]

γ = specific weight of the liquid flowing [F/L^3],

μ = dynamic viscosity of the liquid [FT/L^2],

h = piezometric head [L] (sum of pressure head and elevation),

s = distance measured in the direction of flow [L].

The combination of terms ($Cd^2\gamma/\mu$) is generally called the Darcy permeability, K, and has the units of velocity. The Darcy permeability is a function of

both the medium and the liquid flowing. It can be seen that the Darcy permeability can be separated into two terms, one of which is a function of the liquid flowing, and the other a function only of the medium.

Thus:

$$K = \frac{k_p \gamma}{\mu} \tag{2}$$

where

$$k_p = Cd^2. \tag{3}$$

k_p is called the intrinsic permeability. It is a function of only the porous medium (or more properly the pore geometry of the porous medium).

The preceding equations are only valid when the inertia forces are small compared to the viscous forces. The Reynolds number, R, which is commonly regarded as the ratio of inertia forces to viscous forces, thus becomes an important parameter in defining the limits of the Darcy regime. The Reynolds number is defined as:

$$R = \frac{ud}{\nu} \tag{4}$$

where

u = seepage velocity ($= U/\theta$, where θ is the porosity)[L/t],

ν = kinematic viscosity of the liquid [L^2/T].

Schneebeli [15] has shown that, when the inertia forces are of the same order of magnitude as the viscous forces ($R \cong 1$), laminar flow still prevails, but the Darcy law is no longer valid. From Schneebeli's experiments, the inception of turbulence did not occur until $R \cong 60$.

The seepage velocities can be written for an isotropic medium as:

$$u = \frac{k_p \gamma}{\theta \mu} \frac{\partial h}{\partial x}$$

$$v = \frac{k_p \gamma}{\theta \mu} \frac{\partial h}{\partial y} \tag{5}$$

$$w = \frac{k_p \gamma}{\theta \mu} \frac{\partial h}{\partial z}$$

where u, v, w are the components of the seepage velocity in the three coordinate directions.

2.2 Convective-Dispersion Equation

The equation for the conservation of the dispersing mass is represented mathematically by the convective-dispersion equation [5][14]. For an isotropic-homogeneous medium with one-dimensional uniform flow parallel to the x-axis, the convective-dispersion equation can be written

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = D_1 \frac{\partial^2 s}{\partial x^2} + D_2 \frac{\partial^2 s}{\partial y^2} \tag{6}$$

where the longitudinal coefficient of dispersion is

$$D_1 = \alpha_1 \left| u \right|^{n_1} \tag{7}$$

and the lateral coefficient of dispersion is

$$D_2 = \alpha_2 \left| u \right|^{n_2} \tag{8}$$

s is the concentration of the dispersing substance (= $\frac{\text{mass of substance}}{\text{mass of solution}}$) and α_1, α_2, n_1, n_2 are coefficients which are functions of the pore geometry of a given medium. The influence of molecular diffusion becomes important for very small velocities ($R < 10^{-3}$). For these cases Raimondi, et al. [11] have suggested the relation $D_1 = D_M + \alpha_1 \left| u \right|^{n_1}$, where D_M is the effective molecular diffusion in porous media. The absolute value of the seepage velocity is used in defining the dispersion coefficients since negative dispersion coefficients have no meaning.

Harleman and Rumer [5] have shown that α_1 is approximately 60 times as great as α_2 for a medium consisting of non-uniform sized spheres. For the same medium, they found that n_1 was approximately twice n_2. Their investigations were over the Reynolds number range, $10^{-2} < R < 2$, and for a d = .965 mm.

Other investigators [3][11] have found the coefficient α_1 to be proportional to the average grain size of the medium, i.e.

$$\alpha_1 = \beta d \tag{9}$$

where

d = 50% size from a standard gravimetric sieve analysis [L]

β = constant of proportionality.

n_1 has been found to be very close to unity. When $n_1 = 1.0$, α_1 has the

units of length and β becomes dimensionless. Raimondi, et al. [11] found $\beta = 0.68$ for spheres with diameters of 0.115 mm and 0.545 mm. Their coefficient n_1 was equal to 1.0. Ebach and White [3] found $\beta = 2.92$ and $n_1 = 1.08$ for spheres with diameters of 0.21 mm, 0.99 mm, 3 mm, and 6 mm. Lan, et al. [7] made studies using sieved fractions of sand. Their results were not conclusive but indicated that $\beta \cong 1.0$ when $n_1 = 1.0$. They investigated sand samples where d was equal to 0.70 mm, 0.498 mm, 0.353 mm, 0.228 mm, and 0.160 mm. The coefficients α_2 and n_2 for lateral dispersion have not been studied in as much detail as the coefficients α_1 and n_1.

A relationship appears to exist between the coefficient α_1 and the intrinsic permeability k_p. Combining equations (3) and (9) gives:

$$\alpha_1 = \frac{\beta}{\sqrt{C}} \sqrt{k_p} . \tag{10}$$

Raimondi, et al. [11] state that for random parkings of uniform size particles, $C = 6.15 \times 10^{-4}$ when d is in cm and k_p is in cm^2. This value of C will be compared later to the values obtained in this investigation.

2.3 Longitudinal Dispersion

The flow situation investigated in this report is shown in Fig. II-1 with the proper boundary and initial conditions. For this situation, no gradient of s exists in the y-direction and equation (6) becomes:

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = D_1 \frac{\partial^2 s}{\partial x^2} \tag{11}$$

$s(0,t) = s_o, \; t \geq 0$

$s(x,0) = 0, \; x > 0$

$s(\infty,t) = 0, \; t \geq 0$

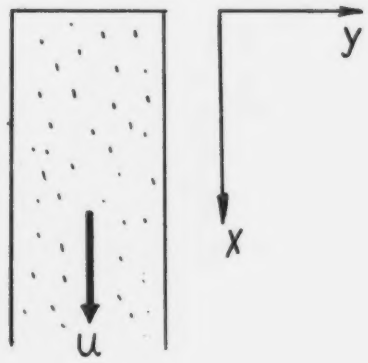


Fig. 1

Schematic Representation of Longitudinal Dispersion

The solution to equation (11) with the boundary and initial conditions indicated in Fig. 1 has been given as in [9][12]:

$$\frac{s}{s_o} = \frac{1}{2}\operatorname{erfc}\left(\frac{x - ut}{2\sqrt{D_1 t}}\right) + \frac{1}{2}\exp\left(\frac{ux}{D_1}\right)\operatorname{erfc}\left(\frac{x + ut}{2\sqrt{D_1 t}}\right). \qquad (12)$$

It has been shown [9][12] that the second term of equation (12) can be neglected when $D_1/ux < .05$. As will be shown later, this is true in the present case. With the second term neglected, equation (12) can be written in dimensionless form as:

$$\frac{s}{s_o} = \frac{1}{2}\operatorname{erfc}\left(\frac{1 - \xi}{2\sqrt{\xi\eta}}\right) \qquad (13)$$

where

$$\xi = \frac{ut}{x}$$

$$\eta = \frac{D_1}{ux}.$$

For the solution given in equation (13) the mean point of the concentration curve ($s/s_o = 0.5$) will move at the seepage velocity, u, in a saturated medium with no dead-end pore spaces or adsorption effects.

Equation (13) can be rewritten in the form

$$\operatorname{erf}\frac{1 - \xi}{2\sqrt{\xi\eta}} = \operatorname{erf}\Phi = 1 - 2\frac{s}{s_o}. \qquad (14)$$

Values of s/s_o can be taken from the experimental results for different values of time. Φ can then be determined from tables of the error function. The quantity $\Phi\sqrt{\eta} = (1-\xi)/2\sqrt{\xi}$ can also be calculated from the experimental results. If $\Phi\sqrt{\eta}$ is plotted versus Φ on rectangular coordinate paper, a straight line results having the slope $\sqrt{\eta}$. From this value of η the coefficient D_1 can be computed. An additional plot of D_1 versus u for each experimental run will yield the coefficients α_1 and n_1.

III. EXPERIMENTAL EQUIPMENT AND PROCEDURE

3.1 Media

Ten different media were investigated. Pertinent information for each medium is summarized in Table I.

In general, the various sizes of media were obtained by sieving a highly mixed batch of material and collecting the separates between two adjacent sieves. The 2 mm spheres were obtained directly from the manufacturer and were produced with a small tolerance on size. The 0.39 mm spheres were also obtained directly from the manufacturer and employed without subsequent sieving. Although the manufacturer of the 0.39 mm spheres indicated that less than ten percent were non-spherical, examination of the beads under a microscope indicated that nearly fifty per cent were non-spherical. For the most part the shape of these non-spherical particles tended to be ellipsoidal but not to such an extent as to be unsatisfactory for the purpose intended.

The sand grains were all sieved from the same bulk sand in order to insure that a similar shape would be obtained from each size. While this is no assurance that the shape of each separate will be identical, the method proved quite satisfactory as viewed from the photographs taken of the media (see Figs. 2 and 3).

The mixture of spheres for series XI was obtained by mixing equal parts of the spheres used for series I through VII. Because of the unnatural makeup of the mixture (i.e. discrete sizes) the usual definition of uniformity (uniformity coefficient = U.C. = d_{60}/d_{10}) is not necessarily a good criterion of the grain size distribution. However, the mean diameter of each separate was taken to represent a typical sieve analysis and the resulting uniformity coefficient was calculated using the mean diameters of each separate.

The mixture of sand for series XII was obtained in a similar manner; however, an additional separate with a mean diameter ranging from 0.59 to 0.71 mm was added to the mixture. The problem of defining the U.C. was again encountered but treated in a similar manner to the spheres.

Some difficulty was experienced in packing the mixture of spheres due to the different particle densities. This problem is discussed in a later section.

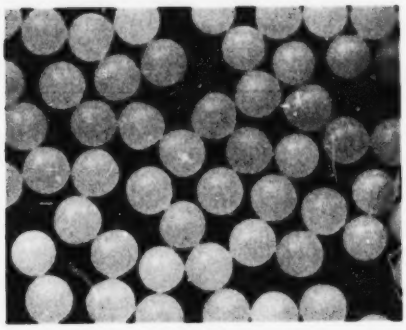
3.2 Column

The tests were performed in a lucite cylinder with an inside diameter of 9.78 cm. (See Fig. 4). The medium was contained in the column by a porous stone plate at the bottom and a 50 mesh screen at

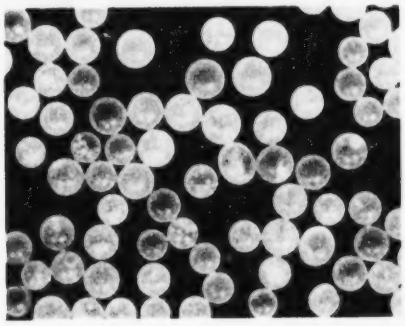
Table I

CHARACTERISTICS OF MEDIA

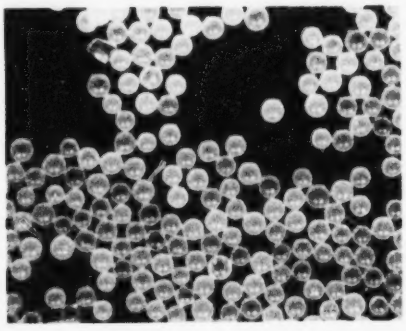
Series	Material	Manufacturer	Shape	Size Distribution (mm)	Uniformity Coefficient	Mean Particle Dia.(mm)	Porosity	Permeability (cm^2)
I	Cellulose Acetate		Sphere	2.0	1.0	2.0	.40	34.3×10^{-6}
II							.37	22×10^{-6}
III							.38	24.5×10^{-6}
IV	Pelaspan 8	Dow Chemical Company	Sphere	1.2 - 1.6	1.15	1.4	.38	15.7×10^{-6}
V	Pelaspan 8	Dow Chemical Company	Sphere	.84 - 1.0	1.08	.42	.37	5.7×10^{-6}
VI	Pelaspan 8	Dow Chemical Company	Sphere	.589-.71	1.10	.65	.36	2.61×10^{-6}
VII	"Superbrite" glass	Minn. Mining and Mfg. Co.	Sphere		1.13	.39	.36	1.03×10^{-6}
VIII	Sand	Jersey Silica Sand Co.	Angular	.84 - 1.0	1.08	.92	.40	4.83×10^{-6}
IX	Sand	Jersey Silica Sand Co.	Angular	1.2 - 1.6	1.15	1.4	.39	9.45×10^{-6}
X	Sand	Jersey Silica Sand Co.	Angular	.4 - .5	1.12	.45	.39	1.10×10^{-6}
XI	Mixture		Sphere	.3 - 2.0	2.75	1.15		not reliable
XII	Sand		Angular	.4 - 1.6	2.15	1.0		2.2×10^{-6}



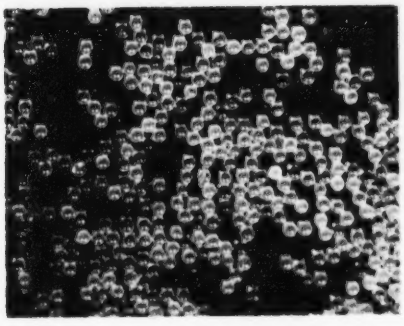
Media I, II, III



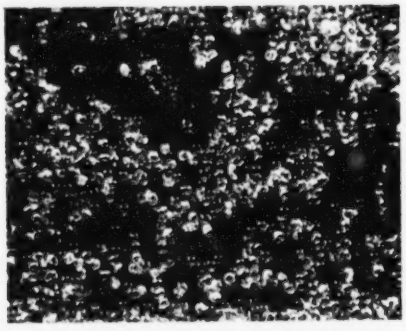
Medium IV



Medium V

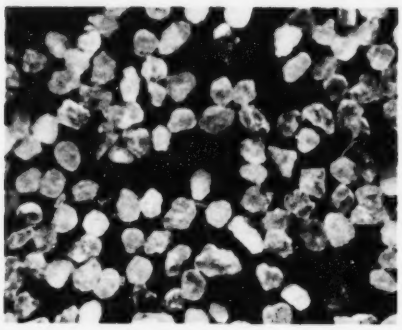


Medium VI

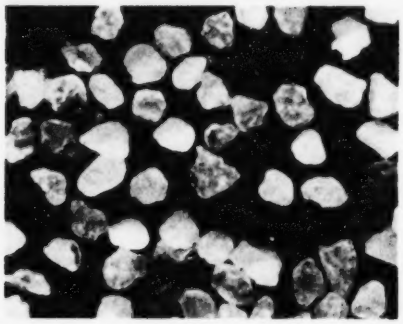


Medium VII

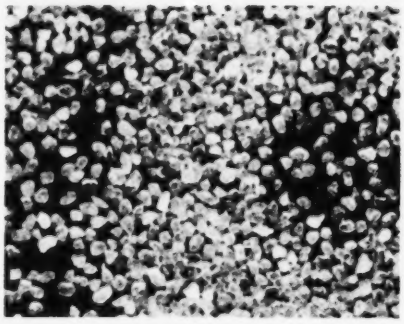
Figure 2. Photograph of Media



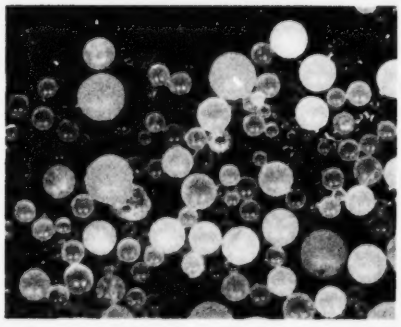
Medium VIII



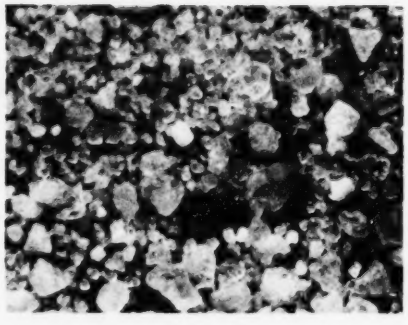
Medium IX



Medium X

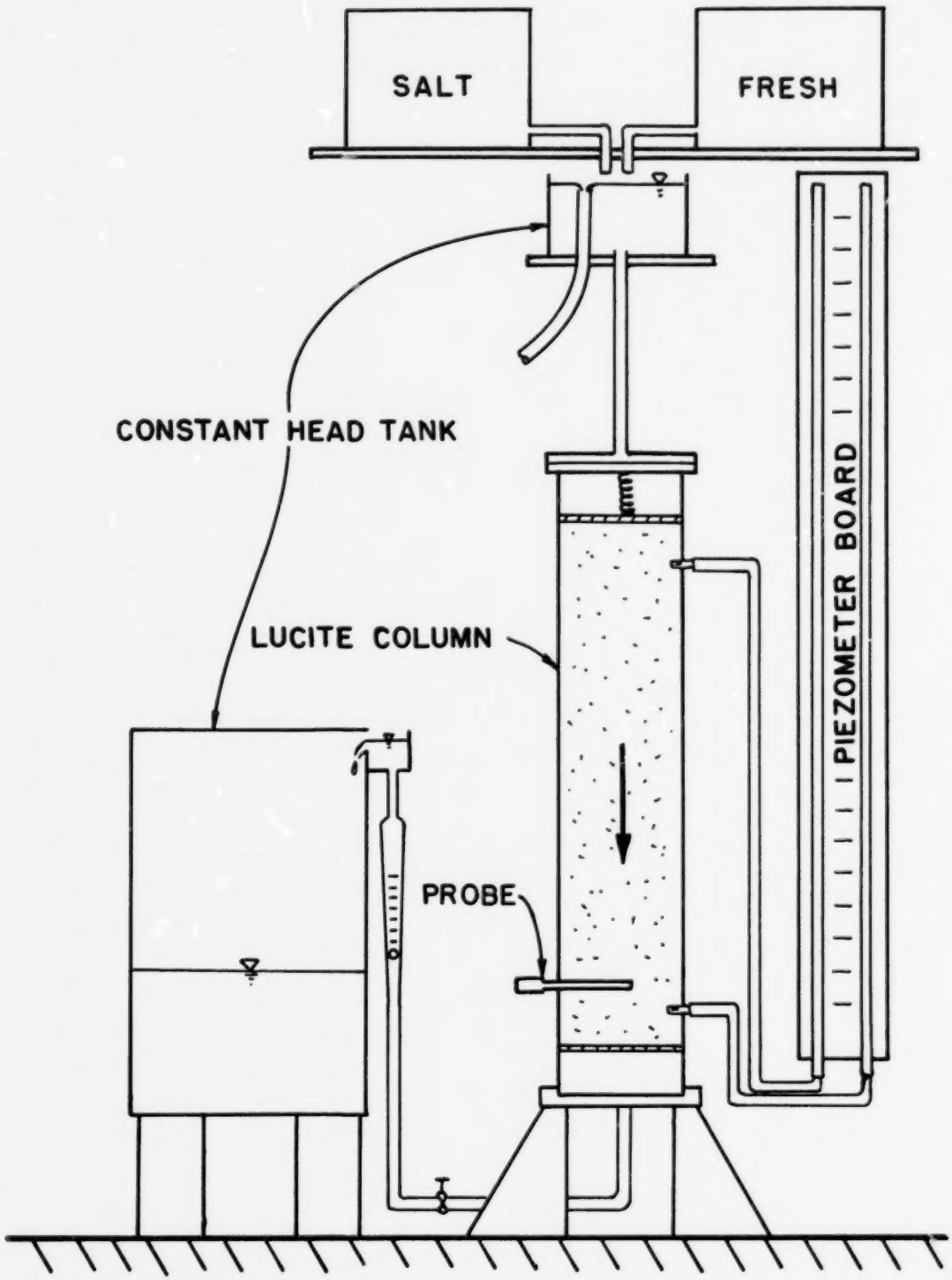


Medium XI



Medium XII

Figure 3. Photograph of Media



SCHEMATIC SKETCH OF EXPERIMENTAL APPARATUS

FIG. 4

the top. The screen was supported by a lucite frame and was a separate unit that could be placed in the column and lowered to any desired position. With this feature it was not necessary to fill the column to any particular level. When a satisfactory packing was achieved, the screen was slid into position and held in place by a compressed spring supported at the opposite end by a cover placed at the top of the column. In this manner the chance of developing void spaces beneath the screen, due to subsequent settling of the medium, was eliminated, since the tension in the spring would force the screen down and keep it in contact with the medium. No settling was noticed, however, but the movable screen did contribute considerably to simplifying the process of filling the column.

Fluid was fed to the column by means of a flexible tube from a constant head tank located approximately one and one-half feet above the top of the column. The constant head tank was fed by two 5-gallon reservoirs, one of which was filled with the tracer solution and the other with fresh water. The flow was collected in a second constant head tank after passing through a control valve and rotameter.

3.3 Instrumentation and Tracer Solution

3.3.1 Tracer

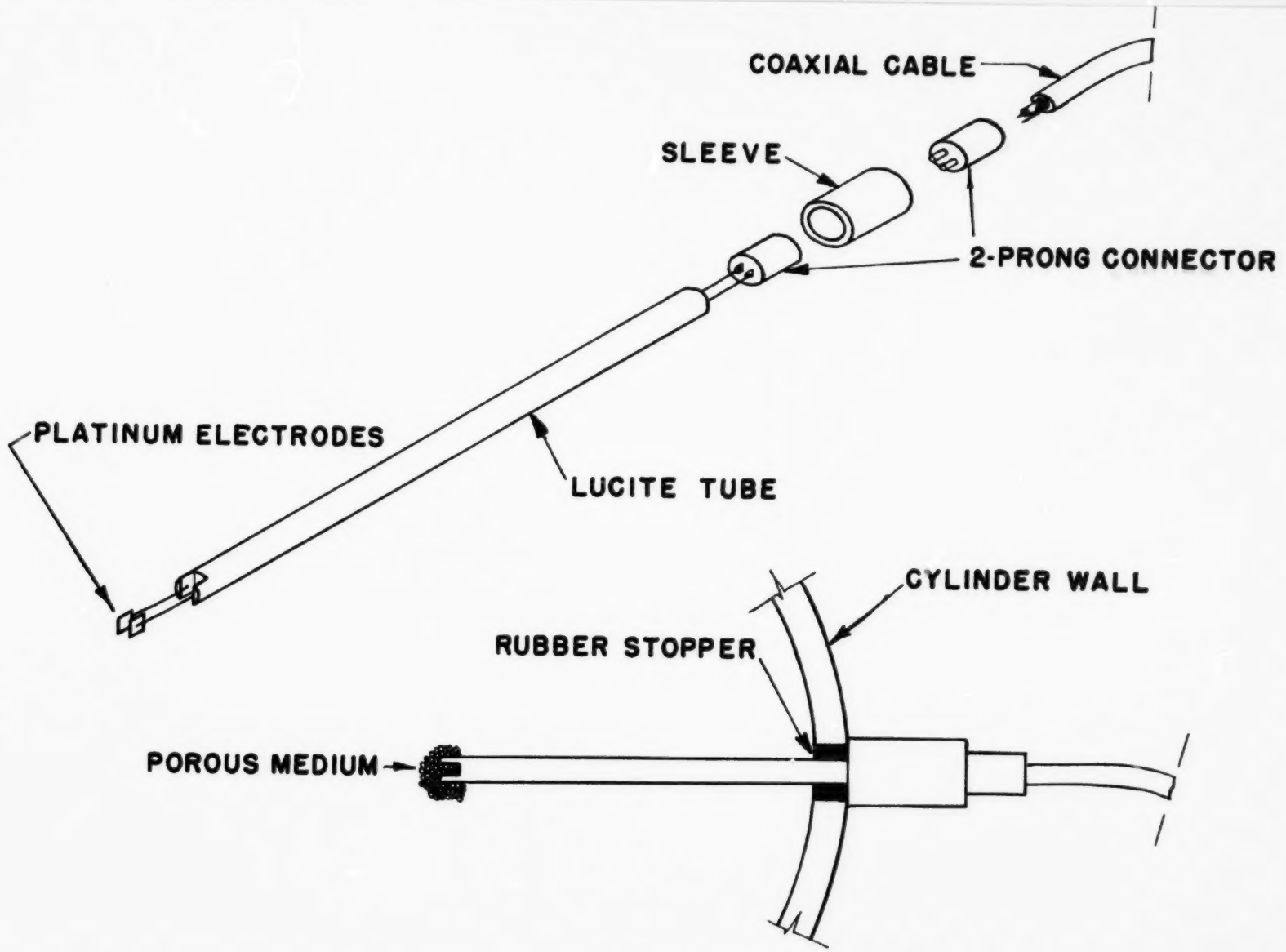
A solution of approximately 0.1 percent by weight of sodium chloride was used as the tracer solution. At this low concentration the conductance of the NaCl solution is known to vary approximately linearly with concentration, thus simplifying the calibration of the probe.

3.3.2 Conductivity Cell

The conductivity cell (see Fig. 5) consisted of a one-quarter inch o.d. lucite tube through which two teflon coated solid copper wires were led. These wires were connected to platinum electrodes. Each electrode was approximately 3 mm on a side. The electrodes were fitted into a slot at the end of the tube and sealed with ethelene dichloride. At the opposite end, the wires were attached to a two-prong connector over which a lucite sleeve was placed and sealed. Small amounts of moisture were seen to collect within the lucite tube from time to time but no effect on the performance of the probe could be detected. The probe was placed in the cylinder through a rubber stopper at approximately 40 cm from the upper screen.

3.3.3 Recorder

A four-channel Sanborn Recorder Model 154 equipped with an independent timer was used to record the signal of the conductivity probe.



SCHEMATIC SKETCH OF CONDUCTIVITY PROBE
FIG. 5

The speed at which the recording paper was fed past the marking stylus could be set for any one of the eight positions ranging from 0.25 to 100 mm/sec. The choice was governed by the velocity at which the dispersed front was expected to pass the conductivity cell. The sensitivity of the amplifier was eight micro-volts per millimeter deflection of the stylus. The frequency response was 0-100 cycles with the standard 2400 cps excitation.

3.3.4 Calibration of Probe

The probe was calibrated in a second column [13]. Solution samples were taken from the column in the vicinity of the probe by means of a one-sixteenth inch stainless steel tube perforated at one end and equipped with a valve at the other. The column was filled with a known concentration of NaCl solution. A sample was taken and the stylus deflection was recorded. The conductance of the samples (volume of each sample was about 1 cc) was measured with a pipette-type conductivity Cel-G1 and type RC Conductivity Bridge, both manufactured by Industrial Instruments, Inc. of Cedar Grove, New Jersey. The accuracy of the bridge was ± 1 percent.

The process was repeated and a calibration curve was determined for the probe. It was discovered that for the small concentrations of NaCl solution being used, the plots of stylus deflection versus concentration were all linear. This simplified the task of analyzing the data since the theoretical solution was in terms of a relative concentration (s/s_o). Thus the relative concentration (s/s_o) could at any time be read directly from the stylus recording by taking a ratio of the stylus deflection at a given time to the maximum stylus deflection when the salt front had passed the probe.

The background concentration of NaCl in the fresh water was eliminated by balancing the probe for zero reading in the fresh water. In this same way the effect on the conductance due to the presence of the porous medium between the electrodes was eliminated. The additional variable of the dependence of conductance on temperature was eliminated providing the temperature remained constant for each individual run. Since this was generally only a matter of minutes, no problem was encountered. Several runs were made in the second column with simultaneous stylus recording and frequent sampling. The curves of s/s_o versus time recorded from the probe signals agreed very well with the corresponding curves plotted from the conductivity analysis of the samples [5].

3.4 Procedure

3.4.1 Packing of Column

The probe was first placed in the test section. The column was packed by filling with a prescribed weight of dry medium and then tapping

the column wall until the medium had settled to a porosity of approximately 37 percent, as determined by measurement of the new length of the packed section. The movable screen was then placed in the column, the retaining spring inserted, and the watertight cover placed on the column and fastened in place. The column was then filled slowly with degased water which was obtained by constantly recirculating tap water and simultaneously spraying it into a vacuum of approximately 20 inches of mercury. The water was fed slowly into the bottom of the column and forced upward through the media, thereby pushing the air before it. The degased water was then passed through the column until all signs of entrapped air were gone. No difficulties with entrapped air were encountered.

3.4.2 Experimental Runs

The tracer solution and fresh water supply were prepared beforehand and allowed to reach room temperature. The fresh water solution was fed into the column from the constant head tank until it had replaced the stagnant solution in the column. The probe was then plugged into the recorder and the stylus balanced at zero. The solution was lowered to the level of the retaining screen, and the head tank and feed system refilled with tracer solution. This established the initial condition, that at $t = 0$, $s = s_o$ for $x \leq 0$.

Simultaneously the timer was turned on and the flow started. When the tracer front had passed through the column, the flow was stopped and the recorder turned off. The procedure was then repeated with fresh water replacing the tracer solution. At a time during the course of the planned tests, permeability measurements were made independently of the breakthrough and elution runs, since the flow in and out of the piezometer taps was a possible means of distorting the dispersion front.

IV. PRESENTATION AND DISCUSSION OF RESULTS

The results of the tests made on uniform particles (Fig. 6 and Fig. 7) show good consistency, the curves of D_1 versus u being clearly defined by the experimental data.

The dashed lines of Fig. 6 indicate the effect of porosity. The lower curve represents a value of porosity of 37 percent while the upper curve represents a value of 40 percent. The runs of series III, made at a porosity of 38 percent are also plotted and in general lie between the curves of series I and II, as would be expected. The values of 37 % and 40 % proved to be the limiting values of the porosity that could be achieved while using the method of packing previously outlined. Attempts at packing with a porosity greater than 41 % resulted in a non-homogeneous and apparently unstable system. In consideration of previous findings it was felt that in subsequent packings if the medium was visually homogeneous the value of porosity would be within the range of 36 % to 40 % and its effect would be minor. This assumption proved to be sound as the porosities of the packings of uniform spheres varied from 36 % to 38 %, and no attempt to correct for variations in porosity was felt necessary.

The results of the experiments are summarized in Table II. The relation between α_1 and d for the uniform sand grains is given by

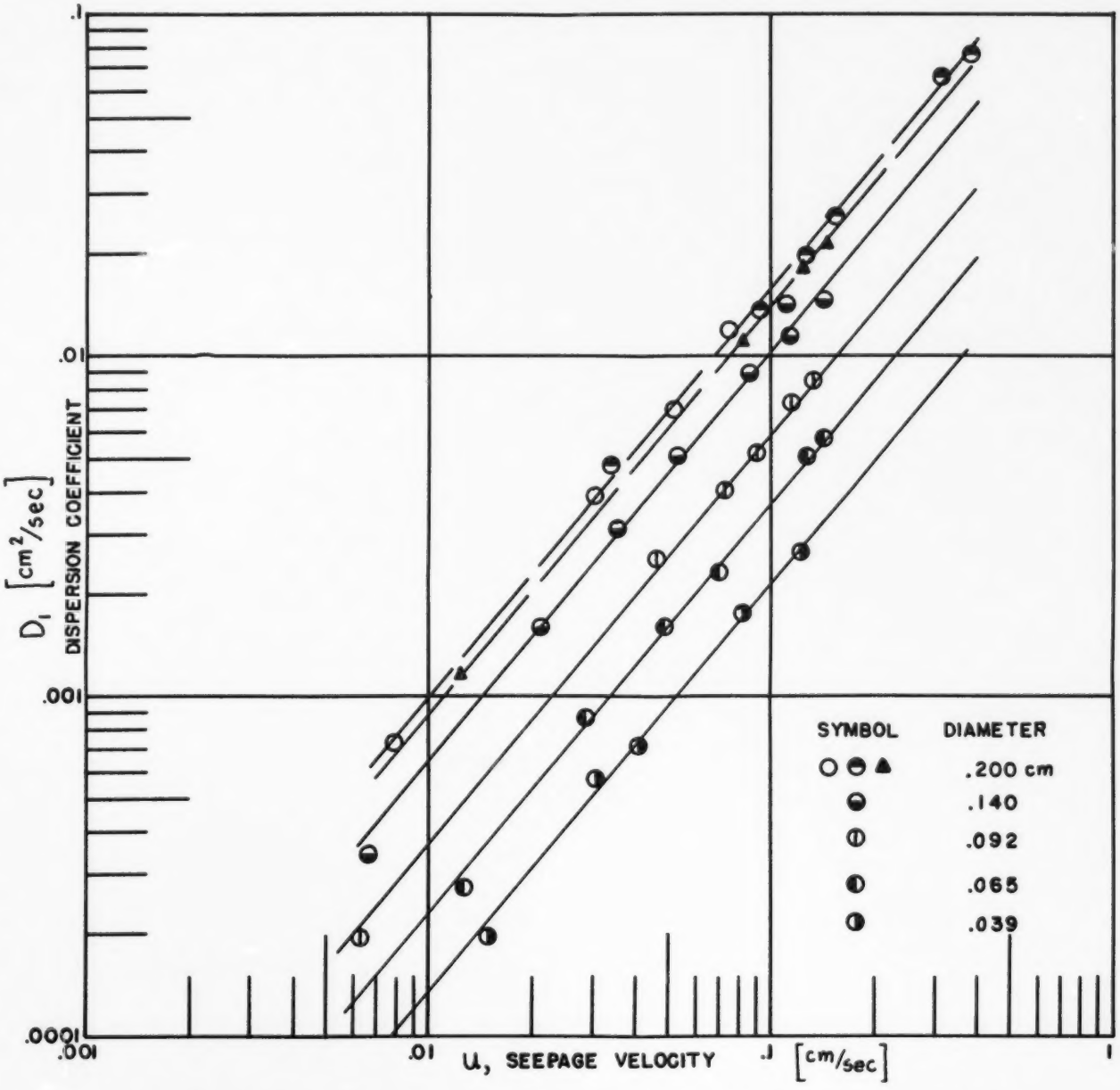
$$\alpha_1 = 1.40\ d \qquad \text{(uniform sand grains)}. \tag{15}$$

For d greater than .05 cm the relation between α_1 and d for the uniform spheres is given by

$$\alpha_1 = 1.40\ d - 0.0315 \qquad \text{(uniform spheres)}. \tag{16}$$

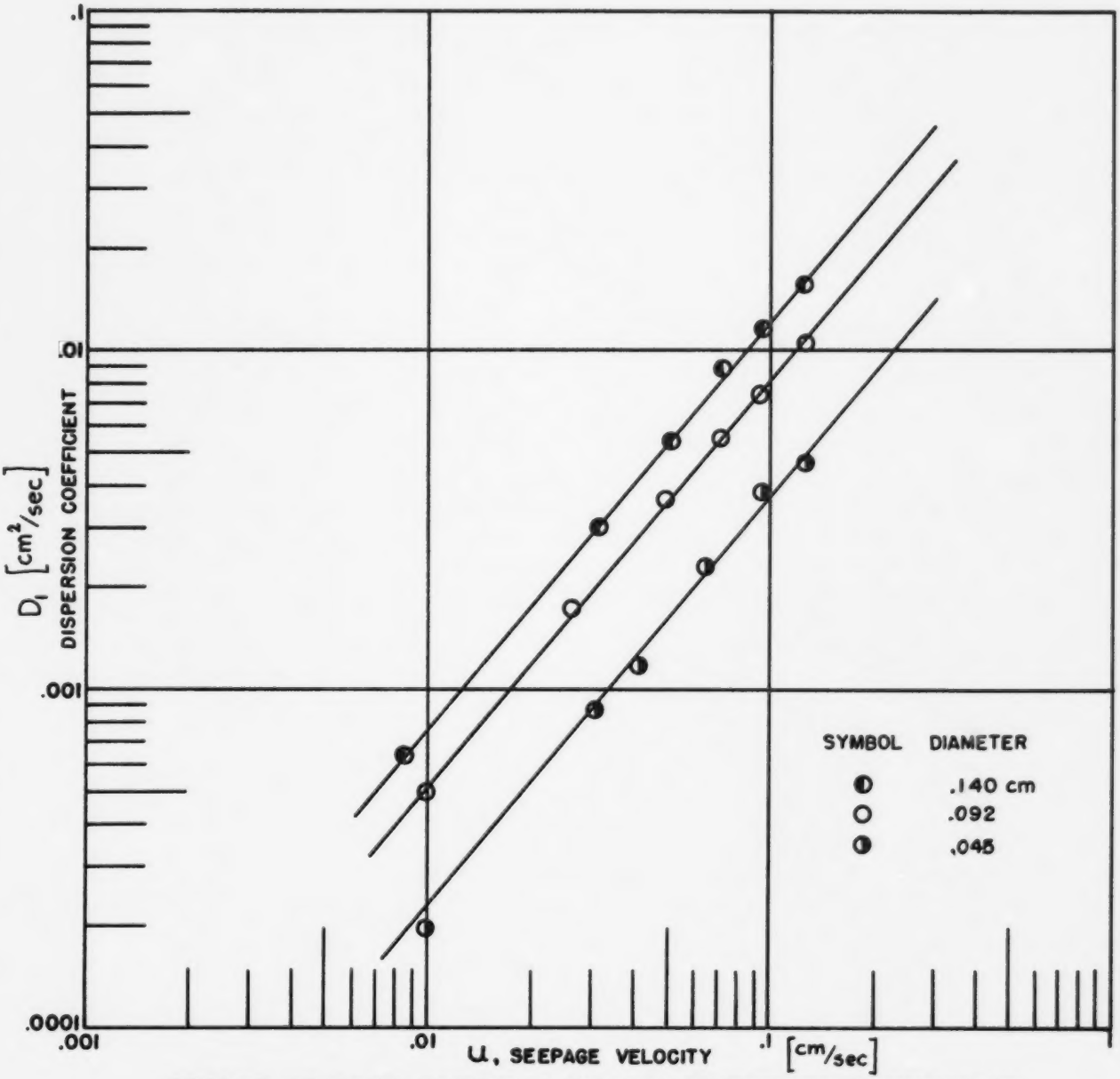
Equation (16) is a straight-line fit to the experimental data. Although the equation indicates that α_1 would be zero for a finite sphere diameter and negative for smaller sphere diameters, such an extension of the curve given by equation (16) would be illogical and not in keeping with the physics of the flow. It is more likely that a transition region exists near the origin and the straight line given by equation (16) would have to gradually curve and intersect the origin. This assumed extension is dotted in Fig. 8. Other data are also plotted for comparison. The slight displacement toward the origin of the data taken for the smallest spheres (d = .039 cm) lends support to the assumed curving of the straight line for $d < .05$ cm.

The tests by Raimondi, et al. [11] on uniform spheres of .0115 cm diameter bear out the above prediction. With the results of their tests, Raimondi, et al. established the relation $\alpha_1 = \beta d$, where $\beta = .68$. It is not certain to the authors that β is actually a constant in this range of



EFFECT OF SPHERE PARTICLE SIZE ON DISPERSION COEFFICIENT

FIG. 6



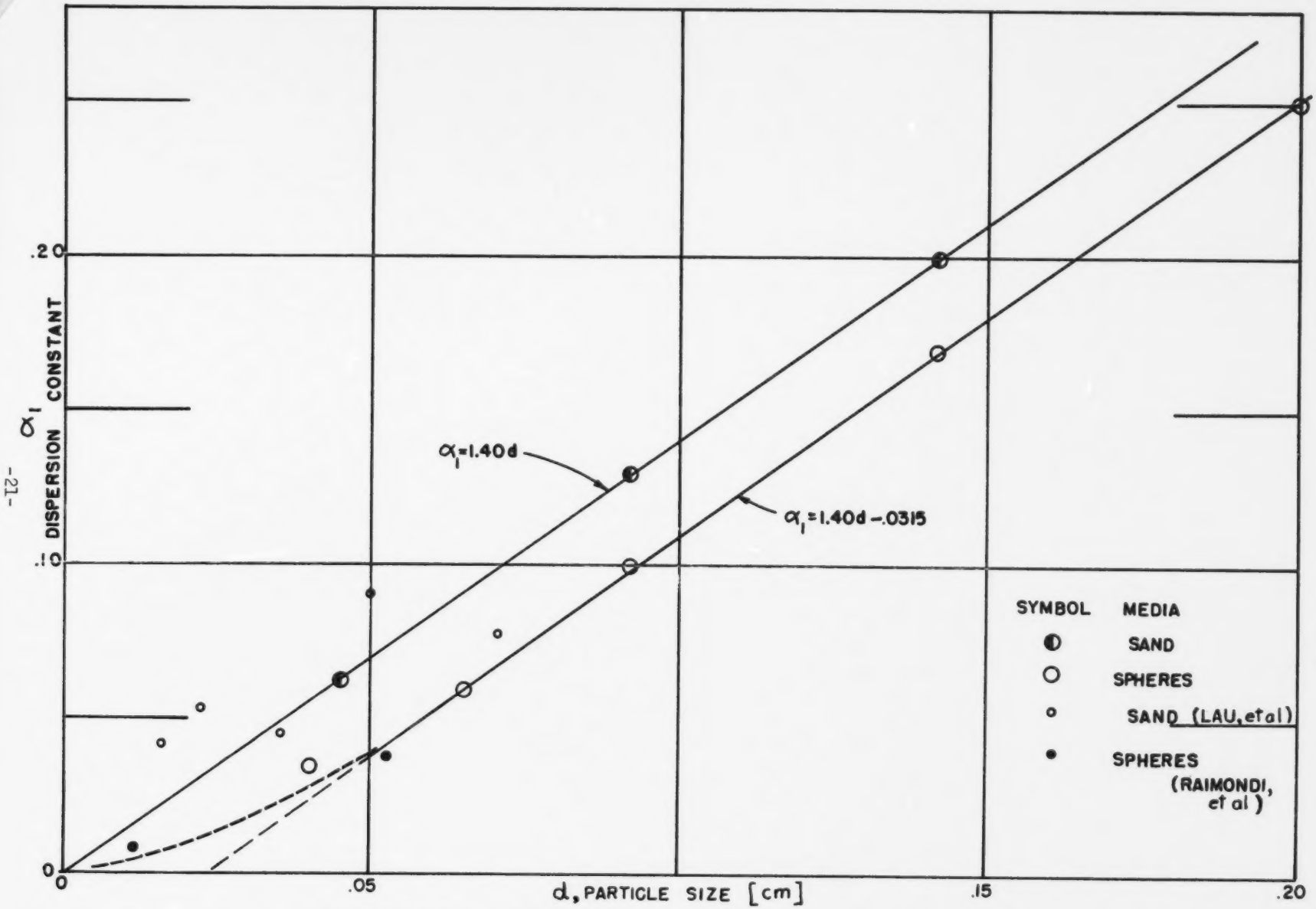
EFFECT OF SAND PARTICLE SIZE ON DISPERSION COEFFICIENT

FIG. 7

Table II

TABULATION OF EXPERIMENTAL RESULTS

Series	Medium	Diameter	a_1	n_1	$< R <$
I	sphere	0.20 cm	.26	1.2	.6 - 6.9
II	sphere	0.20 cm	.245	1.2	.22 - 2.6
III	sphere	0.20 cm	.245-.26	1.2	.144-1.35
IV	sphere	0.14 cm	.170	1.2	.083-1.83
V	sphere	0.042cm	.096	1.2	.053-1.10
VI	sphere	0.065cm	.061	1.2	.074-0.83
VII	sphere	0.039cm	.035	1.2	.052-.43
VIII	sand	0.092cm	.03	1.2	.082-1.05
IX	sand	0.14 cm	.20	1.2	.105-.158
X	sand	0.045cm	.061	1.2	.051-.52
XI	sphere mixture	0.115cm	.045	1.0	.093-.97
XII	sand mixture	0.10cm	.083	1.13	.10 - 1.06



RELATION OF DISPERSION CONSTANT TO PARTICLE SIZE

FIG. 8

d, nor does the limited experimental data indicate so.

For the uniform sand grains, no such break is evident and equation (15) apparently applies over the entire range investigated. Lau, et al. [7] from similar experiments on uniform sand grains obtained comparable results (see Fig. 8). Their scattered results indicate a relation similar to equation (15).

Within the experimental range investigated, n_1 had a constant value of $n_1 = 1.2$ for all media consisting of uniform particles, independent of size and shape.

The validity of equation (3) ($k_p = Cd^2$) is seen in Fig. 9 where the value of C suggested by Raimondi, et al. was used for the best fit curve. The shape effect is seen by the slight upward displacement of the data for the spheres and the slight downward displacement for the sand grains. As a further attempt to correlate the dispersion constant, α_1, with the properties of the media, values of α_1 for each test series were plotted against corresponding values of k_p (see Fig. 10). It is seen that α_1 is given by equation (10). If the value of C obtained from Fig. 9 is used, the quantity $\beta/\sqrt{C}$ becomes equal to 60 for the sand grains and 42 for the spheres. Thus equation (10) becomes for the sand grains

$$\alpha_1 = 60\sqrt{k_p} \qquad \text{(for sand grains)} \qquad (17)$$

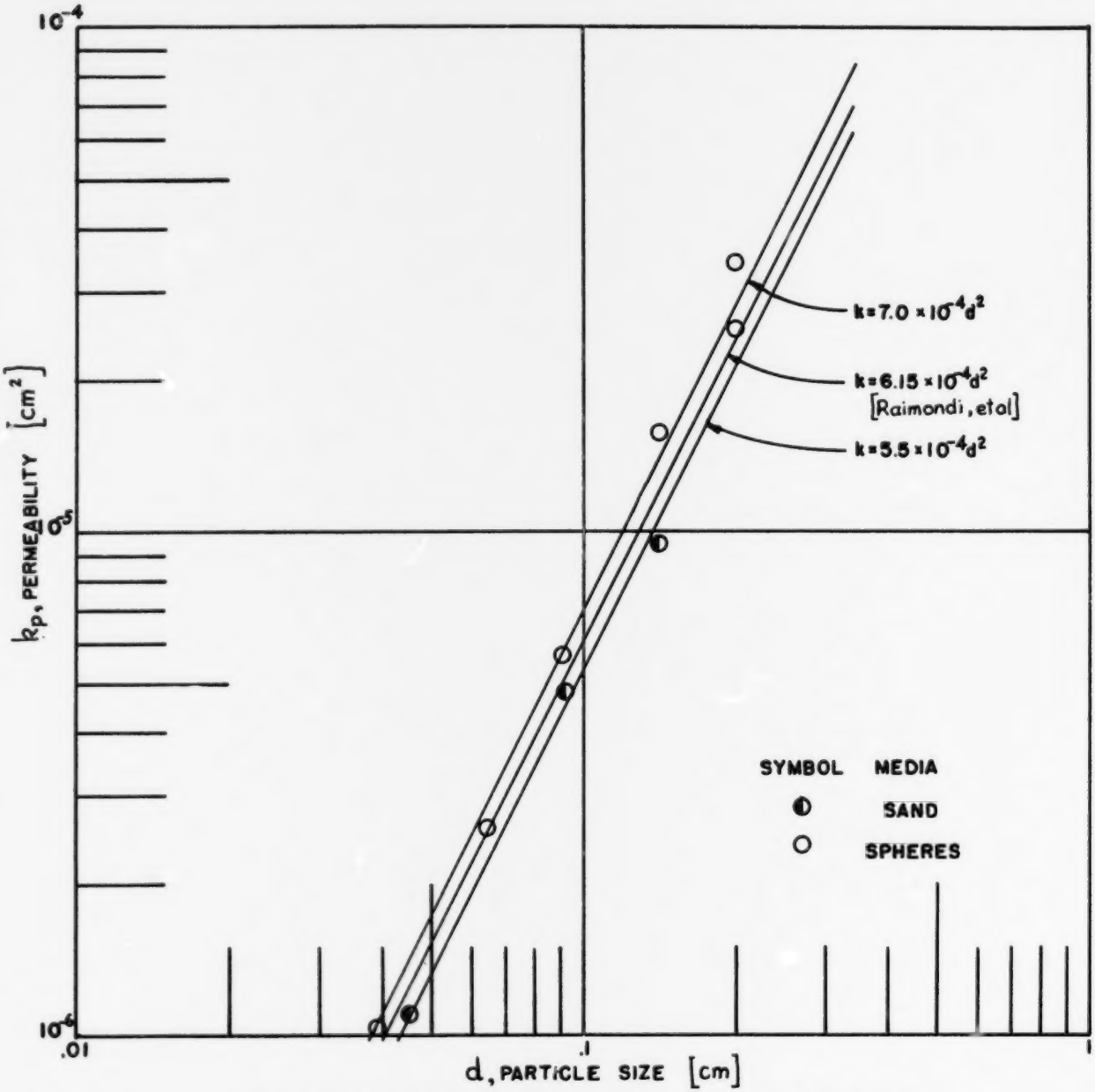
and for the spheres

$$\alpha_1 = 42\sqrt{k_p} \qquad \text{(for spheres).} \qquad (18)$$

Equations (17) and (18) are plotted in Fig. 9 and are seen to be in good agreement with the experimental data.

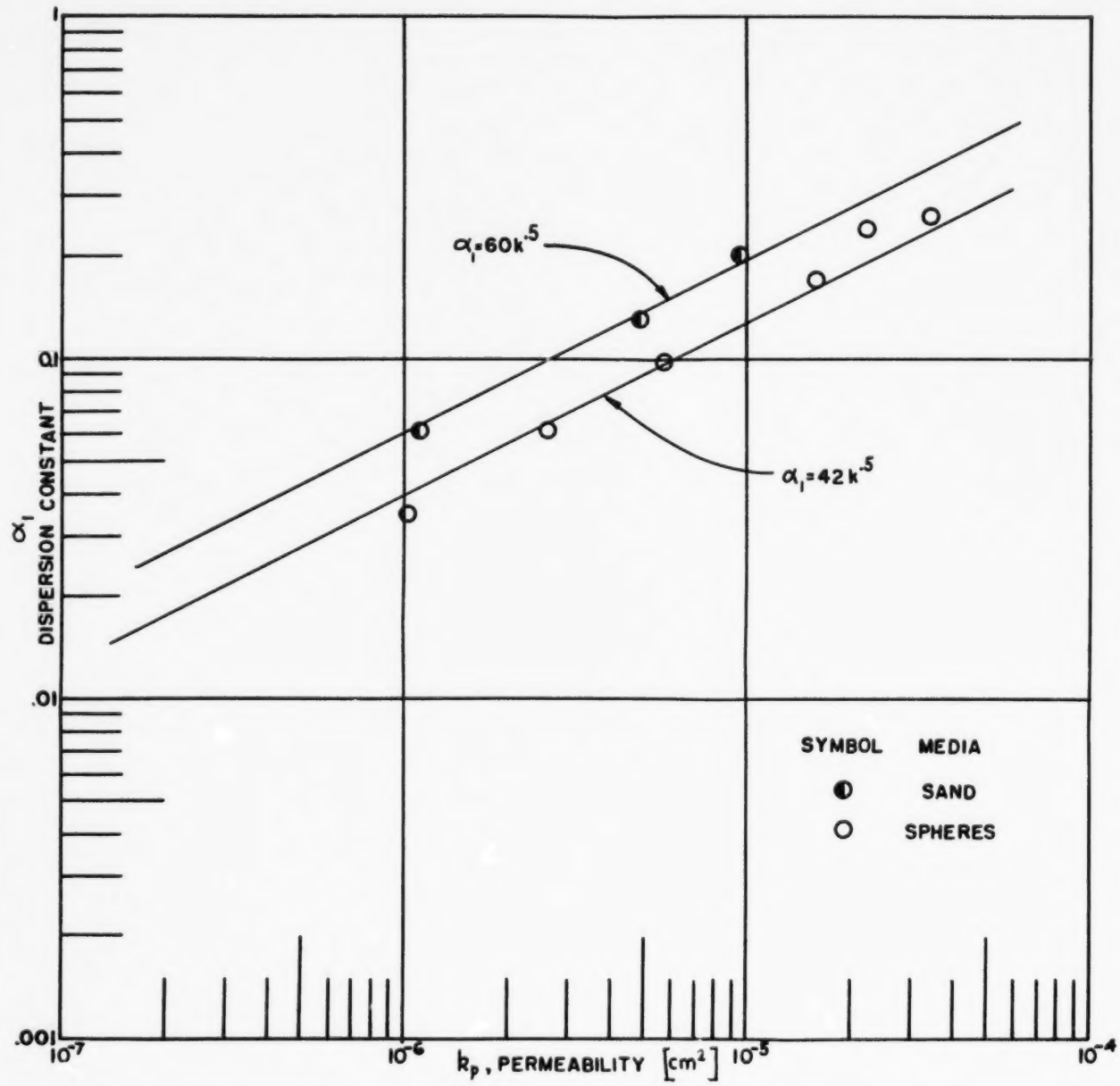
Data taken in the tests performed on media of non-uniform particle size distribution are plotted in Fig. 11. The results indicate a decrease in n_1 with decreasing uniformity (i.e. increasing uniformity coefficient). The curve reflecting the greater non-uniformity is that of the mixture of glass spheres. The uniformity coefficient was taken as 2.75 for the spheres and 2.1 for the sand. Having demonstrated previously that the particle shape had little effect on n_1, the results of the tests on the non-uniform media are significant. It is seen that the distribution of sizes (not their absolute magnitude or shape) is the parameter which has the most effect on n_1.

For the spheres the value of n_1 obtained is close to unity. There is no reason to believe, however, that this is a minimum value of n_1. It is evident from the preceding discussion that the dispersion



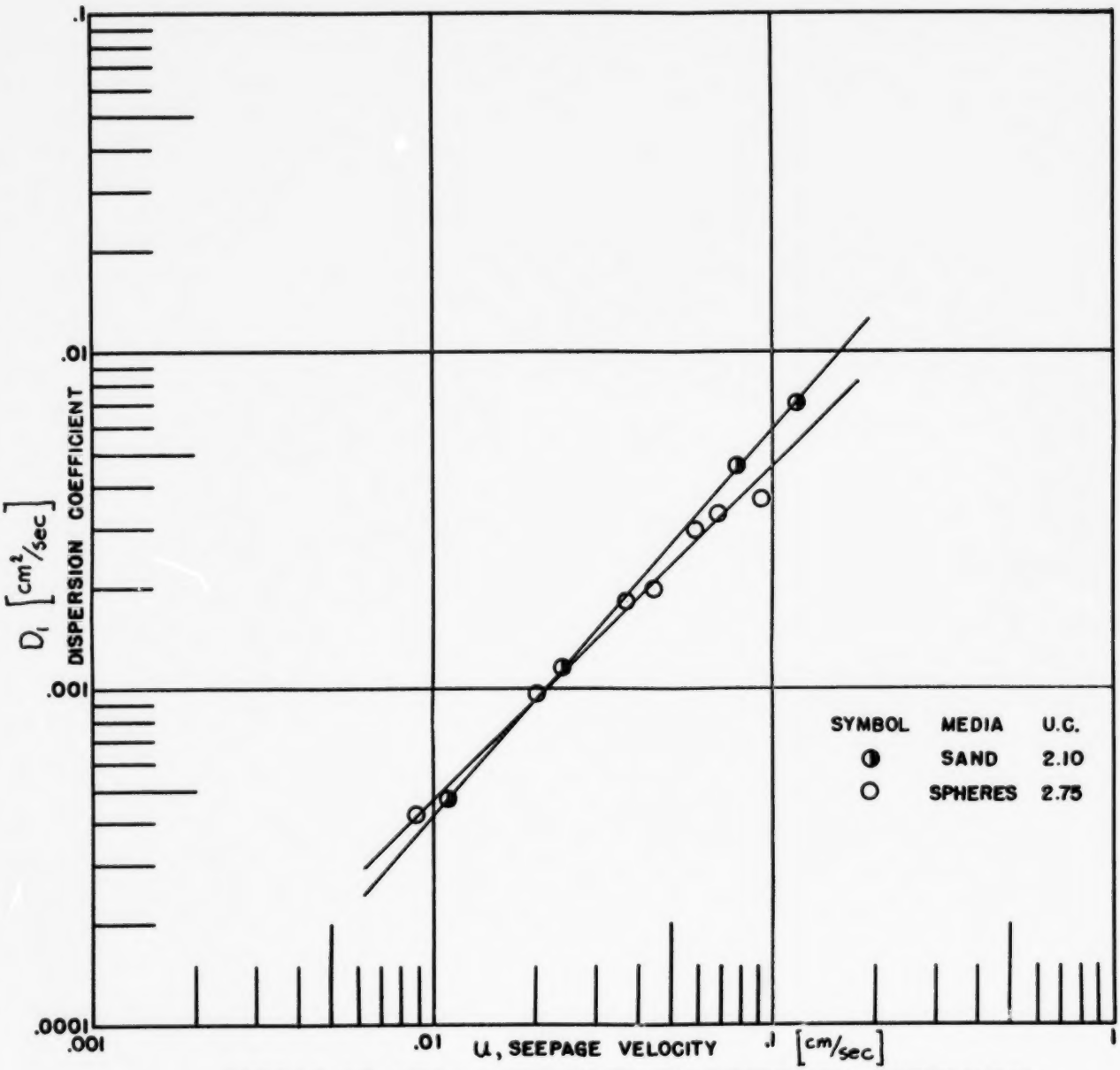
RELATION OF PERMEABILITY TO PARTICLE SIZE

FIG. 9



RELATION OF DISPERSION CONSTANT TO PERMEABILITY

FIG. 10



EFFECT OF MEDIA UNIFORMITY ON DISPERSION COEFFICIENT

FIG. 11

process for non-uniform media is less dependent upon the seepage velocity. If the concepts of circuitry and tortuosity are introduced, some explanation for this behavior can be given. Circuitry is a measure of the difference between the average direction of an individual flow path and the direction of the bulk flow. Tortuosity is a measure of the number of "wiggles" in a flow path. In a complete uniform medium the circuitry would be zero but the tortuosity would be greater than zero. In a non-uniform medium (or one with a distribution of sizes) it has been shown that the tortuosity and circuitry are both greater than zero [16]. As the circuitry increases the flow path of a particle of fluid is also increased (by definition). This phenomenon will tend to reduce the extent to which the velocity influences the dispersion process.

It is noted here that the observation made by Hazen [4] in 1892 that a non-uniform medium will behave hydraulically the same as a uniform medium with grain size equal to the 10% size of the non-uniform medium, appears to be in accordance with the results of the present tests. The correlation of α_1 with the d_{10} for the two non-uniform media gives fair agreement with the correlation of α_1 and d for the uniform media.

V. CONCLUSIONS

Based on the analytical considerations and experimental results of this study the following conclusions are drawn.

(1) The coefficient of longitudinal dispersion is satisfactorily described for laminar flow by the equation:

$$D_l = \alpha_l \left| u \right|^{n_l}$$

when molecular diffusion can be neglected. α_l is primarily a function of the size and shape of the grains. n_l is primarily a function of the distribution of the grain sizes.

(2) It has been demonstrated that α_l is a constant for a given medium in place and is a function of the particle size and shape. The equations describing this relation of α_l to particle size were found to be (for uniform media):

$$\alpha_l = 1.40\ d \qquad \text{(for uniform sand grains)}$$

and

$$\alpha_l = 1.40\ d - .0315 \qquad \text{(for uniform spheres [d > .05 cm])}$$

where d is in centimeters.

(3) For a given porosity and particle shape, α_l varies linearly with $\sqrt{k_p}$, according to the relations:

$$\alpha_l = 60\sqrt{k_p} \qquad \text{(for uniform sand)}$$

$$\alpha_l = 42\sqrt{k_p} \qquad \text{(for uniform spheres)}$$

where k_p is in cm^2. The limited information on non-uniform mixtures of particles also indicates a similar dependency of α_l on $\sqrt{k_p}$, the constant of proportionality being somewhat greater than for uniform media. This is of particular significance since k_p is a more descriptive parameter of the medium as a whole than either particle size or shape and is the more measurable quantity in practice.

(4) The value of n_l is unaffected by changes in particle shape but does reflect the non-uniformity of the medium. As the non-uniformity of the medium is increased the value of n_l can be expected to decrease. It is also a constant for a given medium in place, its order of magnitude being unity.

(5) A complete solution to the determination of α_l can only come from extensive investigation of non-uniform media aimed particularly at

delineating the effect of non-uniformity on the intrinsic permeability, k_p, and the dispersion constant, α_l.

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